Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - I
Subject Code : 4TE01EMT1 **Branch: B.Tech (All)**
Semester : 1 **Date : 14/03/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$ then $x_1 x_2 x_3$..... to ∞ is
(A) – 3 (B) – 2 (C) – 1 (D) 0

b) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
(A) $a=2,\ b=-1$ (B) $a=1,\ b=0$ (C) $a=0,\ b=1$ (D) $a=-1,\ b=2$

c) If $f(x) = \frac{e^x - e^{-x}}{2}$ is continuous at $x=0$, then the value of $f(0)$ must be
(A) 0 (B) 1 (C) 2 (D) 3

d) $\lim_{x\to\infty} x^n e^{-ax}$ (n being a positive integer and $a > 0$) = _____
(A) – 1 (B) 0 (C) 1 (D) None of these

e) The sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$ is
(A) $\log 2$ (B) zero (C) infinite (D) none of these

f) The interval of convergence of the logarithmic series
$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .....\infty$ is
(A) $-1 < x \le 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \le x \le 1$

g) If the two tangents at the point are real and distinct the double point is called
(A) a node (B) a cusp (C) a conjugate point (D) none of these

h) If the power of y are even, then the curve is symmetrical about
(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these

i) The series $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...$ represent expansion of

(A) $\sin x$ (B) $\log(1+x)$ (C) $\cos x$ (D) $\cosh x$

j) If $y=\sin^{-1}x$, then x equal to

(A) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (B) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$

(C) $1-y+\frac{y^2}{2!}-\frac{y^3}{3!}+.....$ (D) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-.....$

k) If $u(x,y,z)=0$ then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) – 1 (C) 0 (D) none of these

l) If $u=f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=0$ (B) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ (C) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$

(D) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$

m) If $f_1=\frac{vw}{u},\ f_2=\frac{wu}{v},\ f_3=\frac{uv}{w};$ then $\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) none of these

n) If errors of 3% in E and – 2% in R are made, then the percentage error in $P=\frac{E^2}{R}$ is

(A) 8% (B) 0% (C) 4% (D) 6%

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$ **(5)**

b) Evaluate: $\lim_{x\to0}\left(\frac{1}{x}\right)^{1-\cos x}$ **(5)**

c) Evaluate: $\lim_{x\to a}\log\left(2-\frac{x}{a}\right)\cot(x-a)$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Using De Moivre's theorem, expand $\sin^8\theta$ in a series of cosines of multiples of θ **(5)**

b) Evaluate: $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$ **(5)**

c) Find the fourth roots of unity and sketch them on the unit circle. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Expand $e^{\sin x}$ as a series of ascending power of x upto x^4. **(5)**

b) Prove that $(1+x)^x=1+x^2-\frac{1}{2}x^3+\frac{5}{6}x^4-.....$ **(5)**

c) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\tan^{-1}n}{n^2+1}$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Test the convergence of the series $\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+.....$ **(5)**

b) Examine the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+.....+\frac{x^n}{n^2+1}+.....$ for convergence using ratio test. **(5)**

c) Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $\left(x-3\right)$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$. **(5)**

b) If $V=\frac{1}{r}$ where $r^2=x^2+y^2+z^2$ show that $V\left(x,y,z\right)$ satisfies Laplace's equation $\frac{\partial^2V}{\partial x^2}+\frac{\partial^2V}{\partial y^2}+\frac{\partial^2V}{\partial z^2}=0$. **(5)**

c) Find the asymptotes of the curve $y^3-x^2\left(6-x\right)=0$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $r=a\left(1+\cos\theta\right)$. **(5)**

b) If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$ then find $\frac{\partial\left(x,\ y,\ z\right)}{\partial\left(r,\ \theta,\ \phi\right)}$. **(5)**

c) If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Trace the curve $x^3+y^3=3axy$. **(5)**

b) Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$. **(5)**

c) Examine the extreme values of $x^2-2xy+\frac{1}{3}y^3-3y$. **(4)**